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On the holographic duals of $\mathcal{N}=1$ gauge dynamics

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Abstract: We analyze the holographic description of several properties of $\mathcal{N}=1$ confining gauge dynamics. In particular we discuss Wilson loops including the issues of a Lüscher term and the broadening of the flux tubes, 't Hooft loops, baryons, instantons, chiral symmetry breaking, the gluino condensate and BPS domain walls.

Keywords: Brane Dynamics in Gauge Theories, AdS-CFT Correspondance, Confinement.

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Contents

1. Introduction

On the route from the original AdS/CFT duality [1, 2, 3] to a holographic description of realistic gauge dynamics, several approaches were used to dualize field theories with $\mathcal{N} = 1$ supersymmetry [4]–[10]. Up to date the holographic description of $\mathcal{N} = 1$ SYM theory has not been written down. However, the models due to Klebanov and Strassler (KS) [8] and Maldacena and Nuñez (MN) [10] made an important step towards this goal.

We will first examine the features of $\mathcal{N} = 1$ gauge theories that can be reliably computed from the holographic dual models. This will be done mainly in the context of the KS and MN models. Among the gauge theory properties that we address are Wilson loops including the issue of the Lüscher term, the broadening of the flux tube, 't Hooft loops, baryons, instantons, chiral symmetry breaking, the gluino condensate, and BPS domain walls. In addition we describe several other possible probes made out of certain brane configurations for which there are no corresponding 4d gauge theory states.

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The second task of this work is to extract from these properties certain guide lines for the construction of other supergravity duals of $\mathcal{N}=1$ confining theories. Based on the very few models in the market, we cannot come up with a precise recipe based on a set of restrictive rules, but rather only with certain "recommendations" for the model builder. An important role in these guide lines is played by wrapped branes. Wrapping of euclidean Dp-brane over $p+1$ cycles, Dp-branes over p cycles, Dp-branes over $p-1$ cycles and Dp-branes over $p-2$ cycles are argued to correspond to instantons, baryons, 't Hooft loops and BPS domain walls. There are also string theory descriptions of $\mathcal{N}=1$ SYM using brane configurations [11], MQCD [12], and brane engineering [13]. In this paper we will concentrate only on holography.

The paper is organized as follows. In section 2 we briefly summarize the supergravity backgrounds proposed by KS and MN. Wilson loops are discussed in section 3. The area law behavior of the two models is extracted from the classical supergravity. We then argue that there exists an attractive Lüscher term, broadening without a roughening phase transition and Regge-like trajectories. In section 4 we discuss baryon configurations in a general confining theory and in the KS and MN models in particular. Section 5 is devoted to the breaking of the $\mathrm{U}(1)_R$ symmetry to Z_{2N} via instantons and the spontaneous breaking down to Z_2. The description of the gluino condensate in terms of the background 3-form is presented in section 6. In section 7 the supergravity configurations that correspond to BPS domain walls are discussed and their tension is computed. Additional brane probes are discussed in section 8. In section 9 we discuss the constraints that each of the gauge dynamics properties imposes on supergravity backgrounds that correspond to $\mathcal{N}=1$ gauge theories.

2. Brief review of the KS and MN models

The original AdS/CFT correspondence can be generalized to $\mathcal{N}=1$ theories in many ways. One way to explicitly break some of the supersymmetries is to place the D3-branes at a conifold singularity [5]. The world-volume gauge theory on the N D3-branes is an $\mathrm{SU}(N)\times\mathrm{SU}(N)$ $\mathcal{N}=1$ superconformal gauge theory with bi-fundamental chiral multiplets A_i and B_i ($i=1,2$) in the $(\bar{N},N)$ and $(N,\bar{N})$ representations of the gauge group. These chiral multiplets transform under the global $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ symmetry as $(2,1)_1$ and $(1,2)_{-1}$. The superpotential is given by

$$\mathcal{W}\sim\mathrm{tr}(A_1B_1A_2B_2-A_1B_2A_2B_1)\,. \tag{2.1}$$

The supergravity dual of this theory is an $AdS_5\times T^{1,1}$ geometry. The correspondence in this background has been worked out in [14, 15].

It is also possible to add M fractional D3-branes, which are D5-branes wrapped on an S^2 of the conifold base [6]. The world-volume theory in such a case is $\mathrm{SU}(N+M)\times\mathrm{SU}(N)$. The addition of M fractional branes explicitly breaks the conformal symmetry. The naive supergravity dual of this theory, found by Klebanov and Tseytlin (KT) [6], has a naked singularity at the origin, and there are non-trivial F_5 and $G_3 = F_3 + iH_3$ profiles related to the number of regular and fractional D3-branes, the dilaton stays constant.

$$\begin{aligned}
\alpha'^{-1} ds^2 &= \frac{u^2}{g_s M \sqrt{\ln(u/u_0)}} dx_{0123}^2 + \frac{g_s M \sqrt{\ln(u/u_0)}}{u^2} du^2 + g_s M \sqrt{\ln(u/u_0)} ds_{T^{1,1}}^2 \,, \\
B_2 &= \frac{3}{2} g_s M (\sin\theta_1 d\theta_1 \wedge d\phi_1 - \sin\theta_2 d\theta_2 \wedge d\phi_2) \ln(u/u_0), \\
\int_{S^3} F_3 &= M \,, \qquad \int_{T^{1,1}} F_5 = N + g_s M^2 \ln(u/u_0) \,.
\end{aligned} \tag{2.2}$$

The two gauge couplings are scale dependent

$$\frac{1}{g_1^2} - \frac{1}{g_2^2} \sim e^{-\phi} \left[\left(\int_{S^2} B_2 \right) - \frac{1}{2} \right] \sim M \ln(u/u_0) \,, \qquad \frac{1}{g_1^2} + \frac{1}{g_2^2} \sim e^{-\phi} = \text{const.} \tag{2.3}$$

This reproduces the logarithmic running of the gauge couplings expected in $\mathcal{N} = 1$ gauge theories.

Klebanov and Strassler [8] have proposed that the naked singularity can be resolved by replacing the conifold with a deformed conifold in which the S^3 part of the conifold base stays at a finite radius near the origin. The logarithmic decrease in the 5-form flux as we flow to the IR was interpreted as a cascade of Seiberg dualities that occur each time one of the coupling constants diverges. In the following we will assume that M is such that at the bottom of the duality cascade we are left with an SU(M) gauge theory. However, this is not a holographic dual of pure SYM, since for $g_s M \gg 1$, where supergravity is a valid approximation, the duality cascade is dense. In other words, there is no finite energy range in which the theory is pure SYM.

The supergravity solution of the deformed conifold is of the following form:

$$ds^2 = h^{-1/2}(\tau) dx_{0123}^2 + h^{1/2}(\tau) ds_6^2 \,, \tag{2.4}$$

where ds_6^2 is the metric of the deformed conifold

$$\begin{aligned}
ds_6^2 = \frac{1}{2}\epsilon^{4/3} K(\tau) \Big[& \frac{1}{3K^3(\tau)} (d\tau^2 + (g^5)^2) + \cosh^2\left(\frac{\tau}{2}\right) [(g^3)^2 + (g^4)^2] + \\
& + \sinh^2\left(\frac{\tau}{2}\right) [(g^1)^2 + (g^2)^2] \Big].
\end{aligned} \tag{2.5}$$

The g^i's are a set of 1-forms parameterizing the conifold

$$\begin{aligned}
\sqrt{2} g_1 &= -\sin\theta_1 d\phi_1 - \cos\psi \sin\theta_2 d\phi_2 + \sin\psi d\theta_2 \,, \\
\sqrt{2} g_2 &= d\theta_1 - \sin\psi \sin\theta_2 d\phi_2 - \cos\psi d\theta_2 \,, \\
\sqrt{2} g_3 &= -\sin\theta_1 d\phi_1 + \cos\psi \sin\theta_2 d\phi_2 - \sin\psi d\theta_2 \,,
\end{aligned}$$

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$$
\begin{aligned}
\sqrt{2} g_4 &= d\theta_1 + \sin\psi \sin\theta_2 d\phi_2 + \cos\psi d\theta_2 \,, \\
g_5 &= d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2 \,, \\
K(\tau) &= \frac{(\sinh(2\tau) - 2\tau)^{1/3}}{2^{1/3} \sinh\tau} \,.
\end{aligned} \tag{2.6}
$$

The function $h(\tau)$ is given by

$$
h(\tau) \sim \alpha \int_\tau^\infty dx \frac{x \coth x - 1}{\sinh^2 x} (\sinh(2x) - 2x)^{1/3} \,. \tag{2.7}
$$

The asymptotic form of this function is $h(\tau \to \infty) \to \alpha\tau e^{-4u/3}$. For large τ we can make a change of variables $u^3 \sim \epsilon^2 e^\tau \sim m^3 e^\tau$. This brings $h(u)$ to the familiar form $h(u) \to \alpha \frac{\ln(u/\epsilon^{2/3})}{u^4}$ which is the KT solution, with $\alpha \sim (g_s M)^2$. F_3 and B_2 are given by

$$
F_3 = M \Big[g^5 \wedge g^3 \wedge g^4 + d[F(\tau)(g^1 \wedge g^3 + g^2 \wedge g^4)] \Big] \,, \tag{2.8}
$$

$$
B_2 = g_s M \Big[f_-(\tau) g^1 \wedge g^2 + f_+(\tau) g^3 \wedge g^4 \Big] \,, \tag{2.9}
$$

where

$$
F(\tau) = \frac{\sinh\tau - \tau}{2\sinh\tau} \,, \qquad f_\pm(\tau) = \frac{\tau\coth\tau - 1}{2\sinh\tau} (\cosh\tau \pm 1) \,. \tag{2.10}
$$

Another possible supergravity dual of pure SYM theory was proposed by Maldacena and Nuñez [10]. They considered N NS5-branes wrapped on an S^2 in the context of 7d gauged supergravity, with boundary conditions that preserve 4 supersymmetries. Their solution is directly related to a non-abelian monopole solution found in [16]. The 10d background is:

$$
ds^2 = dx_{0123}^2 + \tilde{\alpha}' N \left[d\tau^2 + e^{2g(\tau)}(e_1^2 + e_2^2) + \frac{1}{4}(e_3^2 + e_4^2 + e_5^2) \right] \,, \tag{2.11}
$$

$$
e^{2\phi} = e^{2\phi_0} \frac{2 e^{g(\tau)}}{\sinh 2\tau} \,, \qquad e^{2g(\tau)} = \tau \coth 2\tau - \frac{\tau^2}{\sinh^2 2\tau} - \frac{1}{4} \,. \tag{2.12}
$$

The e_i's are a set of 1-form given by

$$
\begin{aligned}
e_1 &= d\theta_1 \,, \\
e_2 &= \sin\theta_1 d\phi_1 \,, \\
e_3 &= \cos\psi d\theta_2 + \sin\psi \sin\theta_2 d\phi_2 - a(\tau) d\theta_1 \,, \\
e_4 &= -\sin\psi d\theta_2 + \cos\psi \sin\theta_2 d\phi_2 - a(\tau) \sin\theta_1 d\phi_1 \,, \\
e_5 &= d\psi + \cos\theta_2 d\phi_2 - \cos\theta_1 d\phi_1 \,, \\
a(\tau) &= \frac{\tau^2}{\sinh^2\tau} \,.
\end{aligned} \tag{2.13}
$$

The 4d gauge coupling is related to the 6d gauge coupling by

$$
\frac{1}{g_{(4)}^2} = \frac{\mathrm{vol}(S^2)}{g_{(6)}^2} = \frac{N e^{2g}}{2\pi^2} \,, \tag{2.14}
$$

where $g^2_{(6)} = (2\pi)^3\tilde{\alpha}'$, so that the 4d coupling is dimensionless. From the asymptotic form of the function $e^{2g(\tau)} \to \tau$, and a change of variables to $\tau \sim \ln(u/m)$, we can see that the 4d gauge coupling runs logarithmically. When flowing to the IR the string coupling becomes of order one, and we must use the S-dual description.

$$\begin{aligned} ds^2 &= e^{\phi}\left[dx^2_{0123} + \alpha' g_s N(d\tau^2 + e^{2g(\tau)}\left(e_1^2 + e_2^2) + \frac{1}{4}(e_3^2 + e_4^2 + e_5^2)\right)\right], \\ e^{2\phi} &= e^{-2\phi_0}\frac{\sinh 2\tau}{2e^{g(\tau)}}, \end{aligned} \tag{2.15}$$

with H_3 replaced by F_3. Note that we keep factors of $\alpha' g_s$ explicit in (2.15), and take $e^{-2\phi_0} \sim g_s N$.

As in the case of the KS background only in the extreme IR is this background supposed to be dual to pure SYM. At higher energies there will be KK states from S^3. In order to decouple these KK states one would like to take the radius of the 3-sphere to zero, $\sqrt{g_s N} \to 0$, but that will produce a region of large curvature, since in both models $\alpha'\mathcal{R} \sim 1/\sqrt{g_s N}$. Both backgrounds are special cases of branes wrapping manifolds with a $R \times S^2 \times S^3$ topology [17, 18]. In the case of N D5-branes and no D3-branes we get the MN solution, and for M D5-branes and N D3-branes with N a multiple of M we get the KS solution.

3. Wilson loops, flux tubes and 't Hooft loops

The long distance quark anti-quark potential is one quantity that is likely to be invariant to the exact dynamics in the UV. From a holographic point of view these backgrounds differ from other supergravity duals of confining theories in that the metric does not have a horizon at a finite radius [19]. The area law in these cases is a consequence of the fact that $f^2(\tau) = g_{tt}g_{xx}(\tau)$ has a minimum at $\tau = 0$, so a fundamental string "prefers" to be on the hyper-surface $\tau = 0$. The finite string tension in both the MN and KS backgrounds is proportional to $f(0)$, and is $\tau_s = \sqrt{g_s N}/2\pi\alpha'$ in the MN model and $\tau_s = m^2/g_s M$ in the KS model. This form of confinement was previously observed in MQCD calculations [12, 20, 21, 22]. Although MQCD is not a holographic description, the Wilson loop can be evaluated in much the same way as in the AdS/CFT.

The action of a fundamental string in the above backgrounds is of the usual form

$$S = \int dx dt \sqrt{f^2(\tau) + g^2(\tau)\tau'^2}\,, \tag{3.1}$$

where $g^2(\tau) = g_{tt}g_{\tau\tau}$. It is easy to see from the metrics in the introduction that we have

$$\begin{aligned} f^2(\tau) &= h^{-1}(\tau)\,, & g^2(\tau) &= 1\,, \quad \text{deformed conifold}. \\ f^2(\tau) &= g_s N\frac{\sinh 2\tau}{2e^{g(\tau)}}\,, & g^2(\tau) &= \alpha' g_s N f^2(\tau) \quad \text{wrapped D5}. \end{aligned} \tag{3.2}$$

In calculating the action of the Wilson loop one has to add also the coupling of the string to the dilaton and to B_{NS}. In the KS model of the dilaton is constant and in MN background there is a non-trivial dilaton. However, in the case of an infinite-strip Wilson loop the gaussian world-sheet curvature vanishes so there is no coupling to the dilaton at leading order in g_s. Both the MN and KS backgrounds have non-trivial B_{NS} profiles. The only components of B_{NS} that can couple to the world-sheet are $B_{01}, B_{0\tau}, B_{1\tau}$. For the backgrounds in question they vanish. Thus, for the two models we investigate here the two additional couplings do not contribute in the leading order in g_s, and the action of the string includes only the Nambu-Goto term (3.1).

We can use the general theorems in [23] to calculate the classical corrections to the linear quark anti-quark potential. The key observation is that in both cases $f(\tau)$ can be approximated near $\tau = 0$ by $f(\tau) = a_0 + a_2\tau^2 + O(\tau^4)$ and $g(\tau) = b_0 + O(\tau^2)$. The resulting potential is therefore

$$E = f(0)L - 2\kappa + O\left((\log(L))^{\beta} e^{-\alpha L}\right). \tag{3.3}$$

The parameters in the potential are [23]

$$\kappa = \int_0^\infty \frac{g(\tau)}{f(\tau)}\left(f(\tau) - \sqrt{f^2(\tau) - f^2(0)}\right) d\tau\,, \qquad \alpha = \frac{\sqrt{2f(0)a_2}}{b_0}. \tag{3.4}$$

Since in both cases $\alpha \neq 0$, the classical correction to the linear potential are exponentially small. This was also the case in other confining backgrounds.

Stringy corrections in the AdS/CFT can come from two sources. One possible source is a correction to the background metric and fields. Since the backgrounds reviewed in the previous section are not group manifolds or coset spaces like $AdS_5 \times S^5$ we really do not have any control on these corrections. Such stringy corrections will change the string tension, but keep the leading linear L dependence. Another source of stringy corrections that becomes relevant in Wilson loop calculations is the fluctuating world-sheet [24]–[27]. Since the classical configuration is a flat horizontal string, these corrections will lead to a quark anti-quark potential with a Lüscher term correction of the form

$$V = f(0)L - \frac{\pi(n_b - n_f)}{24}\frac{1}{L}, \tag{3.5}$$

where n_b and n_f are the number of massless bosonic and fermionic excitations of the string. Close to $\tau = 0$ the metric is effectively $R^{1,6} \times S^3$. The S^3 stays at a finite radius proportional to $\sqrt{g_s N}$. The number of massless bosonic excitations will thus be $n_b = 8$ (or 7 according to [24]) if the distance between the quarks is smaller than the radius of S^3, and $n_b = 5$ if it is larger.

Next we consider the fermionic fluctuations. In [24] it was argued that in the case of the AdS_5 black hole background, which for very high temperature is dual to a 3d Yang-Mills theory, the fermionic modes of a horizontal string are all massive. This is due to the coupling of the fermionic modes to the RR F_5. The KS background also has a RR F_5, but near $\tau = 0$ it vanishes. However, there is a non-vanishing G_3, which for $\tau = 0$ is in the S^3 direction. Since we do not have a candidate string action in this background, we cannot be certain whether the fermion excitations have zero modes or are massive in this background. If the coupling of fermions to the RR 3-form is of the same nature as the coupling to F_5, then the fermionic determinant will be that of massive modes. However, we do have a field theory argument that there are massless fermions. The dual theory in the IR, where this analysis is valid, is pure SYM plus massive KK states. SYM does not have BPS saturated strings. Therefore, we expect the string to have 2 left and 2 right massless fermionic excitations reflecting the fact that it is not invariant under any of the 4 supersymmetries [12]. To summarize, if we combine our arguments about the bosonic and fermionic modes it seems that there is an attractive Lüscher term in the $\mathcal{N} = 1$ models under review, after all even in the $\mathcal{N} = 4$ case one believes that this term does not vanish [26, 25, 24], however at present we cannot show it explicitly.

It was noted that the confining backgrounds such as [19] reproduce some field theory phenomena such as broadening of the flux tube [28] and Regge like trajectories [29]. We will now argue that in the framework of holographic duals these phenomena are generic to a large class of confining backgrounds, and in particular to the two under discussion. The fact that the flux tube broadens already at strong coupling means that one can hope to extrapolate strong coupling results to weak coupling without running into a phase transition. In [30] it was shown that a stringy model of a flux tube in flat space leads to a logarithmic broadening of the flux tube as a function of the distance between the quarks. It was shown in [28] that this is also the case in the AdS_5 black hole metric. The way to calculate this effect is to consider the surface between two concentric Wilson loops with radii R_1, R_2 a distance H apart along the z-axis. The area of the surface is given by

$$S = \frac{1}{2\pi\alpha'} \int r dr \sqrt{f^2(\tau)(1 + z'^2) + g^2(\tau)\tau'^2} \,. \tag{3.6}$$

Substituting the equation of motion for z' into S we get

$$\begin{aligned} S &= \frac{1}{2\pi\alpha'} \int r dr \sqrt{F^2(\tau) + G^2(\tau)\tau'^2} \\ F^2 &= \frac{r^2 f^4}{r^2 f^2 - q^2} \,, \qquad G^2 = \frac{r^2 f^2 g^2}{r^2 f^2 - q^2} \,, \end{aligned} \tag{3.7}$$

where q is an integration constant. The qualitative picture that arises is that if the new F and G lead to confinement then for $R_1 \gg H \gg R_2$ the surface will concentrate at some $\tau = \tau_0$, and there the flat case analysis is valid, and leads to broadening. The two possible scenarios for confinement are: (i) $g(\tau)$ diverges at some τ_{div} while $f(\tau_{\text{div}})$ stays finite. In this case it is clear that $G(\tau)$ and $F(\tau)$ will also have these properties. (ii) $f(\tau)$ has a minimum at $\tau_{\rm min}$. In this case $F(\tau)$ will also have a minimum, although not necessarily at the same τ. We conclude that every confining background will lead to flux tube broadening.

The same reasoning applies to recent work by Janik [29] that uses holography to describe the Pomeron. The results in that case are classically the same as would be expected from a dual string model in flat 4 dimensions. It is only when one considers the stringy corrections to the intercept that the fact that the string is embedded in more than 4 dimensions comes into play. Therefore it seems likely that all confining backgrounds will produce a Regge trajectory.

The "dual" phenomenon of quark confinement is the screening of magnetic monopoles. The later is observed via the 't Hooft loop. Supergravity duals of 't Hooft loops were constructed for models of confining quarks [31, 4]. In those cases one uses un-wrapped D1-branes, or D2-branes in type IIA [32, 33]. However, one can easily check that these objects are not adequate in the MN and KS models, since a D1-brane will have a non-vanishing string tension. It is not clear to us what field theory object it corresponds to in the 4d theory as we further discuss in section 8.

Instead it was argued [8, 10] that the holographic description of gauge theory monopoles in both backgrounds are fractional D1-branes. As pointed out in [8] one can only prove that they are not confined. The action of a D3-brane wrapped on an S^2 will have the same general form as the action of the string, but with the functions $f^2(\tau)$ and $g^2(\tau)$ multiplied by the volume of the S^2. In both cases this volume vanishes at $\tau = 0$ so the monopoles have vanishing string tension and are not confined.

4. Baryons

The baryon vertex of $\mathcal{N} = 4$ SU(N) SYM is dual to a D5-brane wrapping S^5 in $AdS_5 \times S^5$. This vertex is connected to N points on the boundary of the AdS_5 with N fundamental strings [34]. The analysis of a configuration of this type in dual supergravity models of confining gauge theories [31, 35] showed their stability and led to a computation of the baryonic mass.

In the holographic models of $\mathcal{N} = 1$ there are two types of "building blocks" for the construction of the baryonic vertex: (i) wrapped D3-brane over S^3 and (ii) D5-branes that wrap the compact 5 dimensional manifold, for instance the base of the deformed conifold in the KS model. There are two types of baryons to which one needs to provide supergravity duals. The first type is similar to the original $AdS_5 \times S^5$ baryon, namely a baryon constructed from external quarks. The second type is a baryon built from the "elementary particles" of the theory.

Prior to the discussion of the assignment of wrapped branes to the two different types of baryons, we first follow the simplified method[1] of [31] for a general back-

[1]An improved approximation which also incorporates the gauge field on the brane was used

ground with N units of flux. The basic configuration is that of a baryonic vertex located at a radial position τ_0, connected with N strings to N "quarks" distributed in a symmetric way on a circle of radius L on the boundary. The baryonic vertex is taken to be a wrapped Dp-brane around some p-cycle at τ_0.

Let $S_{\mathrm{D}p}(\tau_0)$ denote the action of the wrapped Dp-brane at τ_0. It has the following form

$$S_{\mathrm{D}p}(\tau_0) = \frac{N}{(2\pi)^p \alpha'^{(p+1)/2}} \int d^p\theta e^{-\phi} \sqrt{\det g(\tau_0)}\,, \tag{4.1}$$

where g is the induced metric on the wrapped brane, and the θ's are coordinates on the p-cycle the brane wraps. The N strings attached to this vertex have a Nambu-Goto action. The total action is $S_{\mathrm{D}p} + NS_{F1}$. The difference between the baryon and Wilson loop calculations is that in the baryon case the strings have one end not on the boundary but rather on the baryon vertex. This produces a surface term in the string action, which must be equal to the force the baryon vertex applies on the strings for the whole system to be static.

$$N \frac{g^2(\tau_0)\tau_0' \delta\tau_0}{\sqrt{f^2(\tau_0) + g^2(\tau_0)(\tau_0')^2}} = \partial_{\tau_0} S_{\mathrm{D}p}(\tau_0)\delta\tau_0\,, \tag{4.2}$$

where the τ_0' denotes the value of $d\tau/dx$ at τ_0. From this condition one can get the radius and energy of the baryon by the same procedure as for the quark anti-quark potential

$$\begin{aligned} L &= \int_{\tau_0}^{\infty} d\tau \frac{g(\tau)}{f(\tau)} \frac{1}{\sqrt{\beta f^2(\tau) - 1}}, \qquad \beta = \frac{g^2(\tau_0)/f^2(\tau_0)}{g^2(\tau_0) - (\partial_{\tau_0} S_{\mathrm{D}p}(\tau_0))^2}\,, \\ E &= N \int_{\tau_0}^{\infty} d\tau \frac{\sqrt{\beta} f(\tau) g(\tau)}{\sqrt{\beta f^2(\tau) - 1}} - N \int_0^{\infty} d\tau g(\tau)\,. \end{aligned} \tag{4.3}$$

In confining backgrounds both integrals receive most of their contributions from the region $\beta f^2(\tau) \approx 1$, and $f(\tau)$, $g(\tau)$ in the KS and MN backgrounds are regular functions with a minimum at $\tau = 0$ so the energy of the baryon is linearly proportional to its size L and to N.

Now we would like to check which wrapped branes correspond to what baryons. In the conformal case [5] there are two types of baryons built from composites of N A particles and N B particles [36]. The corresponding brane configurations involves wrapping a D3-brane over two supersymmetric cycles of minimal volume associated with constant (θ_1, ϕ_1) or (θ_2, ϕ_2). The dimensions of these operators were computed and were shown to be identical to the field theory dimensions. In a similar manner one can write down two type of baryonic operators in the KS model [37]. In the UV the baryons are built from a composite of $\frac{N+M}{M}$ (for $N \bmod M = 0$) singlets of

in [35]. This type of calculation can also be applied for the present case, however to deduce the basic properties of the system the simplified approach suffices the SU(N) group factors which themselves are composites of N A particles (or B particles). Replacing N with $P = N - nM$ in this construction produces the baryons after n steps of the Seiberg duality. At the end of the cascade, where $P = 0$, there are no matter fields in the left over theory with SU(M) gauge group and hence there are no baryons composed from "dynamical quarks", but one can still construct a baryon made out of "external quarks".

In [37] a proposal for the dual configuration was suggested based on the following configuration. In the asymptotic regime it is a set of $\frac{M+N}{M}$ D3-branes wrapped on the S^3 with M string ending on each D3-brane in such a way that the total of $N+M$ strings end on their other side on a single D5-brane that wraps the five-cycle. The number of strings attached to any D3-brane and the D5-brane is obviously the amount of the G_3 and F_5 flux. This constitutes the required baryon operators in the region where the flux of F_5 is $N + M$ and that of G_3 is M. In the region where the former is $p+M$ and the latter is still M, a similar construction holds with $\frac{M+P}{M}$ wrapped D3-branes connected to a single wrapped D5-brane with $M + P$ strings. Finally in the IR region where $p = 0$ the M strings stretching out of the wrapped D3-brane cannot be attached to the 5-brane since now the F_5 flux vanishes and hence can end only on M external quarks.

5. Instantons, and the breaking of $\mathrm{U}(1)_R$ to Z_{2N}

Instantons in $\mathcal{N} = 1$ SYM theory break the $\mathrm{U}(1)_R$ symmetry to Z_{2N}. To exactly identify the dual of the field theory instanton we need an object with an action of the field theory instanton,

$$S_{\text{inst.}} = \frac{8\pi^2}{g^2_{YM}(\mu)} + i\theta_{FT}\,. \tag{5.1}$$

In the large N limit the breaking of the $\mathrm{U}(1)_R$ to Z_{2N} cannot be seen in the isometries of the classical background. In order to see this breaking one has to consider instanton probes, and to show that their action is invariant only under the Z_{2N} symmetry. The supergravity dual of the field theory instanton can in general be a combination of a D(-1) and a wrapped world-sheet of a D1/F1-string.

In the MN background the SU(N) instanton was argued to be a D1-brane wrapping the S^2 and another S^2 inside the S^3 which is defined by $\theta = \theta_1 = \theta_2$ and $\phi = \phi_1 = \phi_2$ [10]. In the UV one uses the S-dual background and the D1-brane is replaced with a fundamental string. Let us see what is the action of such a string. For simplicity we will carry out all calculations in the UV, and use the singular MN metric. The value of the induced $B_{\theta\phi}$ on the specified 2-cycle is $B_{\theta\phi} = -\frac{N\psi}{2}\sin\theta$. Therefore the imaginary part of the world-sheet action of the F1-strings comes from the WZ term

$$\frac{1}{2\pi}\int_\psi d\theta d\phi B_{\theta\phi} = b - N\psi\,. \tag{5.2}$$

where b is some integration constant. This flux should be identified with the phase θ_{FT} in the field theory action so it is clear that only Z_{2N} rotations of this phase $\psi \to \psi + 2\pi k/N$ leave the path integral invariant.

Now let us compute the real part of the action, S_R of this configuration. By wrapping the string world-sheet over the same cycle we get a Nambu-Goto action of the form

$$S_R = \frac{1}{2\pi\tilde{\alpha}'}\int d\theta d\phi\sqrt{g} = 2N\sqrt{e^{4g(\tau)} + \frac{1}{2}e^{2g(\tau)} + \frac{1}{16}} \tag{5.3}$$

In the UV, where $e^{2g(\tau)} \sim \tau$, the first term is dominant and we get $S_R = 2Ne^{2g(\tau)} = 4\pi^2/g_{YM}^2$. The instanton action in the UV is indeed proportional to g_{YM}^{-2}, but there are corrections as can be seen from (5.3). There does not seem to be a 4d object that is dual to the D(-1) (see also section 8).

The field theory instanton in the KS background is a D1-brane wrapped on a 2-cycle of the conifold in order to break the U$(1)_R$. However, a wrapped string will not give the correct real part of the instanton action. The field theory instanton should be a combination of a wrapped string and a D(-1). The motivation for this ansatz comes from (2.3). The sum of the gauge couplings is proportional to the dilaton, which couples to a D(-1), while the difference is proportional to the flux of B_2 on the 2-cycle we argued was responsible for the breaking of the U$(1)_R$ symmetry. The action of a D(-1) in this background is constant and real, so D(-1)'s are not responsible for the U$(1)_R$ symmetry breaking. We will again carry out all calculation using the singular background in the UV (the KT background). The value of the induced RR 2-form over the 2-cycle $\theta = \theta_1 = \theta_2, \phi = \phi_1 = -\phi_2$ is the same as B_2 was in the MN background, $C_{\theta\phi} = \frac{N\psi}{2}\sin\theta$, and therefore will produce the same phase. The real part of the wrapped D1-brane action is

$$S_R = \frac{1}{2\pi\alpha' g_s}\int d\theta d\phi\sqrt{\det(g + 2\pi\alpha' B)} \sim M\sqrt{\ln^2(u/u_0) + b\ln(u/u_0)}\,, \tag{5.4}$$

where b is some constant. In the UV the first term is dominant and we indeed get that the action is proportional to the difference of gauge couplings as in (2.3). Again, there are corrections as we flow to the IR. Note also that neither of the backgrounds reproduces the correct factor of $8\pi^2$ in (5.1).

The Z_{2N} non-anomalous R-symmetry is further spontaneously broken to Z_2. In field theory this breaking of the discrete symmetry is manifested in the existence of N degenerate vacua. These are the minima of the superpotential. The order parameter associated with this breaking is the gluino condensate, $\langle \mathrm{tr}\,\lambda\lambda\rangle$.

In the KS and the MN backgrounds the spontaneous breaking of Z_{2N} down to Z_2 is caused by the blow-up of the S^3 at the conifold singularity. The existence N degenerate vacua (M in KS) translates in the supergravity picture into the fact that there are only N discrete backgrounds that are truly non-singular [10]. Since at the origin the S^2 which the F1 or D1 world-sheet wraps is contractable the flux through this cycle has to be an integer multiple of 2π. Asymptotically, for $\tau \to \infty$, the flux has this property. One has to find directions along which the S^2 can be transported to the origin, $\tau = 0$, without any change of the flux (or a change of the form $2\pi n$). Because the angle ψ is trivially fibered over S^2, the directions in question will be along constant $\psi = \psi_0$. It is clear that $\psi_0 = 0$ is one such value. The radial τ component of the 3-form is proportional to $\sin\psi$ so that along $\psi_0 = 0$ the wrapped world-sheet can be transported to the origin $\tau = 0$ without any change of the flux. One can do the same for any $\psi_0 = 2\pi k/N$, $k = 0, \ldots, N-1$. The radial τ component of the 3-form is now proportional to $\sin(\psi - \psi_0)$, which again vanishes allowing the S^2 to be transported to the origin. The N vacua are labeled by the phase of the gluino condensate as we will see in the next section.

6. The gluino condensate

In this section we will concentrate on the gluino bilinear, $\mathrm{tr}\,\lambda\lambda(x)$. Most of our arguments will be based on the KS model. It was argued in [8] that in the UV where $N \gg M$ the anomalous dimensions of operators like $\mathrm{tr}\,\lambda\lambda(x)$ are only of order $O(M/N)$ or less, although the theory is not conformal. If at the bottom of the cascade we are left with pure SYM then the dimension of $\mathrm{tr}\,\lambda\lambda(x)$ is also protected in the IR. This is because $\mathrm{tr}\,\lambda\lambda(x)$ is one of a set of operators on the field theory side that are not supposed to get any anomalous dimensions. These operators are the components of the so-called anomaly multiplet. The dimensions of the operators in this multiplet are protected by virtue of the fact that the highest component is the trace of the energy-momentum tensor, which is a conserved current. The lowest component is the gluino bilinear which has dimension 3.

We would like to associate one polarization of $C_2 = C_2^{RR} + iB_2^{NS}$ with the operator $\mathrm{tr}\,\lambda\lambda(x)$. The value of $G_3 = dC_2$ at $\tau \to \infty$ is (we write only the polarizations along $T^{1,1}$)

$$G_3 \to \frac{M}{2}(1+\tau e^{-\tau})g^5\wedge g^3\wedge g^4 + \frac{M}{2}(1-\tau e^{-\tau})g^5\wedge g^1\wedge g^2 + \frac{ig_sM}{2}\tau e^{-\tau}g^5\wedge(g^1\wedge g^3 + g^2\wedge g^4)\,. \tag{6.1}$$

If we subtract the asymptotic value of G_3, which has nothing to do with chiral symmetry breaking (in fact it is the same as in the KT solution) we get

$$\Delta G_3 = \frac{M}{2}\tau e^{-\tau}\omega_3\,, \qquad \omega_3 = \left[g^5\wedge(g^3\wedge g^4 - g^1\wedge g^2) + ig_s g^5\wedge(g^1\wedge g^3 + g^2\wedge g^4)\right]. \tag{6.2}$$

This is the polarization we would like to associate with $\mathrm{tr}\,\lambda\lambda(x)$. Furthermore, it can be seen from (2.6) that the corresponding polarization of C_2,

$$C_2 = -\frac{M}{2}\tau e^{-\tau}\omega_2\,, \qquad \omega_2 = \left[(g^1\wedge g^3 + g^2\wedge g^4) + ig_s(g^1\wedge g^2 - g^3\wedge g^4)\right], \tag{6.3}$$

transforms by a phase when $\psi \to \psi + \delta\psi$, in agreement with fact that $\mathrm{tr}\,\lambda\lambda(x)$ is charged under the broken $\mathrm{U}(1)_R$.

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For large τ one can change variables to $u = \epsilon^{2/3} e^{\tau/3}$. We will use the identification made in [8] that the deformation parameter ϵ is related to the 4d mass scale as $m \sim \epsilon^{2/3}$. Thus, we get that close to the boundary ΔG_3 is of the form

$$\Delta G_3 = \frac{M}{2} \frac{m^3}{u^3} \ln \frac{u^3}{m^3} \omega_3 \,. \tag{6.4}$$

In a conformal theory (AdS) this is almost the behavior we would expect of a scalar operator of dimension 3 that has a VEV. We know that in the field theory dual the Z_{2M} symmetry is broken by the deformation of the conifold to Z_2. We see from (6.4) that this is in agreement with a non-zero $\langle \mathrm{tr}\, \lambda\lambda \rangle \sim M m^3$.

The two point correlation function of the gluino bilinear can be derived by considering the effective action for the polarization of G_3 mentioned above.

$$\begin{aligned} C_2 &\to C_2 + y(x,u)\omega_2 \,, \\ G_3 &\to G_3 + y(x,u)\omega_3 + dy(x,u) \wedge \omega_2 \,, \end{aligned} \tag{6.5}$$

where $y(x,u)$ is the perturbation with non-vanishing boundary values $y_0(x_1)$ and $y_0(x_2)$. There could and probably are large mixings between these modes and other modes, but this simplified calculation is only intended to show that the 2-point function is space-time independent.

Substituting these perturbations into the relevant part of the supergravity action

$$\int d^4x du \sqrt{-g} \left[G_3 G_3^* + \frac{1}{2} \left(F_5 - \frac{1}{2i}(C_2 \wedge G_3^* - C_2^* \wedge G_3) \right)^2 \right], \tag{6.6}$$

and integration over u will not yield a kinetic term of the schematic form $dy_0(x_1) \times dy_0(x_2)$. The field theory interpretation of this is the well-known result that correlation functions of lowest components of chiral superfields are space-time independent. It remains to be shown that a mass term of the schematic form $m^6 y_0(x_1) y_0(x_2)$ does originate. It can be seen from (2.5) that the supergravity action is indeed proportional to $\epsilon^{12/3} \sim m^6$, through $\sqrt{g}$, and that the quartic term in (6.6) will produce a term quadratic in y_0.

The two point function of the gluino bilinear with its complex conjugate, $\langle \mathrm{tr}\, \lambda\lambda(x)\, \mathrm{tr}\, \bar{\lambda}\bar{\lambda}(y) \rangle$ will receive contributions from glueball exchange [38, 39]. Operators like $\mathrm{tr}\, \lambda\lambda(x)$ are only the lowest members of a tower of KK states [15]. One must keep in mind that pure $\mathcal{N} = 1$ SYM does not have these operators in its spectrum. If one can show that there are no mixings between these supergravity modes and the lowest mode, then in calculating 2-point functions of point operators one can effectively decouple these KK states. In calculating 2-point functions of extended operators like Wilson loops one has to remove the contributions from such states by hand [40], because the string world-sheet can couple to all such operators.

Gluino condensation in the MN background is somewhat more subtle. We can repeat the same analysis that was carried out in the KS background, namely that the difference between the value of G_3 in the singular and the deformed solution is the dual of the gluino bilinear. However, since the UV is dual to some twisted 6d theory we do not have a good understanding of why this is the dual supergravity mode. Another argument, which will be presented in the next section, is that G_3 couples to 5-branes, that play the role of domain walls.

7. BPS domain walls

Let us first briefly review the basic properties of the domain walls. $\mathcal{N}=1$ supersymmetric gauge theories, which are characterized by a discrete set of vacua, generically admit BPS domain wall configurations that interpolate between the inequivalent vacuum states. The domain walls preserve half of the $\mathcal{N}=1$ supersymmetries. In the field theory picture such BPS-saturated walls satisfy first-order differential equations, which follow in a straightforward manner from the requirement of preserving two out of four global supersymmetries. Denoting the superpotential and the Kähler metric by $W(A^i)$ and $g_{j\bar{i}}(A^i, \bar{A}^{\bar{j}})$ respectively, where A^i are the chiral superfields describing the low energy effective action, and taking the coordinate normal to the wall to be x_3, the condition takes the following form

$$\left(Q_\alpha - i e^{i(\Delta W)_{\rm arg}} \sigma^3_{\alpha\dot{\alpha}} \overline{Q}^{\dot{\alpha}}\right) |\text{wall}\rangle = 0\,, \tag{7.1}$$

where $(\Delta W)_{\rm arg}$ is the argument of the difference of the superpotential between the two vacua. This condition translates into the following first order differential equation [41]

$$\frac{dA^i}{dx_3} = e^{i(\Delta W)_{\rm arg}} g^{i\bar{j}} \frac{\partial W^*}{\partial \bar{A}^{\bar{j}}}. \tag{7.2}$$

The tension of the BPS domain wall is exactly determined by the difference between the superpotential values in the two vacua connected by the wall. In the $\mathcal{N}=1$ SYM theory, the superpotential, which acts as a central charge for domain walls, is related to the gluino condensate, so a BPS domain wall has tension [41]

$$\mathcal{T}_{DW} = \frac{N}{8\pi^2} \left|\Delta\langle tr\lambda\lambda\rangle\right|. \tag{7.3}$$

In the large N limit the domain wall tension is linear in N. The trace yields a factor of N but the phase difference of two vacua is proportional to N^{-1}.

In $\mathcal{N}=1$ theories the effective superpotential is so constrained by the twin requirements of the holomorphy and flavor symmetry that one can completely determine its exact form [42, 43, 44]. Unfortunately, no such constraints apply to the effective Kähler function of the theory which controls the kinetic energies of the fields. In fact, due to the lack of knowledge of the latter the story of BPS domain walls in $\mathcal{N}=1$ is not fully established. The Kähler function is under much better control in theories with $\mathcal{N}=1$ which are derived by mass perturbation of the Seiberg-Witten $\mathcal{N}=2$ SYM models [45, 46].

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BPS domain walls were analyzed also in the context of MQCD [12, 45, 47]. In that formulation the vacua correspond to various Riemann 2-cycles that the M5 wraps and the BPS-saturated domain walls correspond to the supersymmetric 3-cycles that interpolate between those Riemann surfaces. So far analytic expressions for such configurations were not discovered [45]. However, it was shown [12] that the parameters of the models can be chosen in such a way that the string tension does not depend of N, whereas the domain wall tension is linear in N.

Recently, domain walls were discussed in the context of the $\mathcal{N} = 1^*$ model [4, 48]. In this model there is a very rich structure of vacua including both confining and Coulomb phase vacua. In the former case, namely, configuration that interpolate between massive vacua the BPS equations again involve the superpotential as well as the Kähler function expressed in terms of the degrees of freedom of the low energy effective theory. It is thus clear that those configurations cannot be determined using the techniques of [48]

We now turn to the description of the BPS domain walls in the supergravity duals of theories with $\mathcal{N} = 1$. Domain walls are attributed in these models [8, 10] to D5-branes wrapping the S^3. Since the volume of S^3 is minimized at $\tau = 0$ the D5-brane will prefer to be in this region. This is in agreement with the fact that only in the IR does the theory have N vacua. In the notation of MN the domain wall spans the coordinates x_0, x_1, x_2, e_i, $(i = 3, \ldots, 5)$, so that it corresponds to turning on a shift of the 3-form G_3 which has components along the S^3 and depends on x_3 in such a way that once one pass from $x_3 = -\infty$ to $x_3 = \infty$ there is a change of the flux of B_2 through S^2 in the S^3 as was explained in section 4. A domain wall interpolating between adjacent vacua corresponds to a single brane since it generates a jump of one unit in the flux whereas k coincident D5-branes are associated with a domain wall connecting vacua that differ by k units of flux. Both the KS and the MN backgrounds are of the form $R^7 \times S^3$ near $\tau = 0$, with the S^3 having radius proportional to $\sqrt{g_s N}$. Therefore, a D5-brane wrapping the S^3 and x_0, x_1, x_2 will give a domain wall with tension

$$\mathcal{T}_{DW} = \frac{1}{\alpha'^3 g_s} \int_{S^3} \sqrt{g} = \frac{N}{\alpha'^{3/2}} (g_s N)^{-1/2} \,. \tag{7.4}$$

The S^3 of the conifold is a supersymmetric cycle [36]. It is thus plausible that the wrapped D5-branes constitute BPS saturated states. This can be also examined from the low energy field theory on the brane. It was argued recently [49] that the $\mathcal{N} = 2$ (4 supersymmetries) inherited from the $\mathcal{N} = 1$ in four dimensions is broken down to $\mathcal{N} = 1$, namely two supersymmetries, by a level N Chern-Simons term. In the present type IIB setup it is a result of a $\int C_2 \wedge F \wedge F = \int G_3 \wedge CS(A)$ term in the D5 world-volume action, which after integration over the S^3 turns into a CS term at level N.

	wrapped S^2	wrapped S^3	wrapped $S^3 \times S^2$	un-wrapped
$D(-1)$	/	/	/	instantons, (a)
F1-strings	instantons (a)	/	/	Wilson loop
D1	instantons	/	/	(b)
D3	't Hooft loop	baryons	/	(c)
D5	4d world-volume	domain walls	baryons	/
(p, q) 5-branes	world-volume	(d)	(d)	/
D7	/	(e)	(e)	/

Table 1: Brane probes.

Several additional properties of the field theory BPS domain wall can be understood in the supergravity language:

(i) Crossing a domain wall made out of k wrapped five-branes in the MN model implies changing the value of ψ_0 by $\Delta\psi_0 = 2\pi k/N$ which is caused by a change of the flux on the S^2 by k units. Since for $k = N$ the value of ψ_0 is the same on both sides of the walls it is anticipated that in this case the domain wall should fade away. Indeed it was shown in [10] that in the 7d form of the background the wrapped NS5-brane is charged under the 3-form potential. Due to the fact that it couples to the $\mathrm{SU}(2)_R$ gauge field, the N wrapped five branes can be replaced by an infinitely large gauge instanton and hence disappear.

(ii) Since a fundamental string can end on a (wrapped) D5-brane, and since the role of a flux tube connecting quark anti-quark is played by a fundamental string, it is clear that in the supergravity picture of a domain wall flux tubes can end on it.

(iii) Another feature of the domain wall is that a baryon can be dissolve in it [4]. Recall that the baryon vertex is a wrapped D3-brane over the S^3 with N strings attached, so it can be embedded in the wrapped D5-brane so that there are N strings ending on the wall.

8. Other supergravity brane probes

Most of the $\mathcal{N} = 1$ gauge theory features discussed in the previous sections were associated with certain brane probes. This raises the question of whether all stable brane probe configurations can be attributed to certain properties of the dual 4d gauge theories. To address this question we summarize in the following table the configurations that have already been discussed together with other possible wrapped and un-wrapped brane probes. We make certain comments about possible interpretation of the latter in the gauge theory picture.

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(a) As was discussed in section 4, a D(-1) brane combined with a wrapped D1-brane play the role of the field theory instanton in the KS model. In the MN background D(-1)'s do not seem to be connected with 4d instantons because they do not have the right action. It is possible that they correspond to some state in the 6d theory. Whereas in MN the F1 world-sheet wrapped over S^2 is associated with the instantons in the UV (and the D1 in the IR), it is not clear to us what is the role of wrapped F1 probes in the KS model.

(b) Unlike their role in the $N = 1^*$ model [4], the un-wrapped D1-branes cannot play the role of the 't Hooft loops connecting an external monopole anti-monopole pair. This configuration can be thought of as a dimensional reduction of the D5/D1 system (6d instanton) on S^2. In the case where the D1 is wrapping the S^2 it is a 4d instanton (a). If it does not wrap S^2, and we hold the ends of the D1 on the boundary, then from the gauge theory perspective it is a point-like object with an action linear in N. The gauge theory interpretation of such objects is not clear to us. In the MN model a D1-brane in the IR is a F1-string in the UV, where the metric is flat. Therefore, such a string will stay at the boundary. It might be identified with a string in the 6d theory.

(c) D3-brane. We start with a configuration of a D3 ending on a D5, and then dimensionally reduce on S^2. This configuration is a monopole in the 6d theory. If the D3 is wrapped on S^2 it has an interpretation of a 4d monopole. If the D3 does not wrap S^2, and we hold one of its ends on the boundary, it looks like an infinite tension (external) domain wall from the 4d perspective [50].

(d) (p,q)-strings and 5-branes. In $N = 1^*$ theory [4] the un-wrapped (p,q) strings serve as the probes of the (p,q) vacua [4, 51]. Since this theory is a daughter theory of the $\mathrm{SL}(2,\mathbb{Z})$ invariant $\mathcal{N} = 4$ theory these strings can be obtained from F1-strings by an $\mathrm{SL}(2,\mathbb{Z})$ transformation. In $\mathcal{N} = 1$ SYM there are no remnants of the $\mathrm{SL}(2,\mathbb{Z})$ structure and indeed the (p,q) vacua do not exist, so it is not clear what is the interpretation of (p,q) strings both un-wrapped and those that wrap the S^2. If one does not find any reason to dismiss this kind of probes in the supergravity picture, it may hint that the vacua of the $\mathcal{N} = 1$ holographic dual of the supergravity backgrounds discussed, are characterized by additional order parameters apart from the gluino condensate. Recall that in the perturbed $\mathcal{N} = 2$ Seiberg-Witten theory in addition to the expectation values $U = \langle \mathrm{Tr}\,\Phi^2 \rangle$ there are also monopoles (or dyons) condensates [45, 46]. It might be that the instanton like objects, associated with the wrapping of the (p,q) strings, are related to these additional order parameters. In a similar manner, it might be that the wrapping of the (p,q)

5-branes over the S^3 are associated with the domain walls profiles of the various monopole and dyon fields [45, 46] that interpolate between the values of these fields at the different vacua. One probably has to treat the "baryons" associated with the wrapping of (p,q) 5-branes over the $S^3 \times S^2$ on the same footing.

(e) Wrapped D7-brane over the S^3 are points on the S^2 and as such do not have 4d duals. By wrapping D7-branes over $S^2 \times S^3$ we get a membrane from the 4d point of view. Again one may want to interpret them as domain walls but together with the wrapped D5 branes and the unwrapped D3 branes it is clear that supergravity offers too rich a set of domain wall candidates, and presumably only the former has the appropriate 4d field theory interpretation.

9. Guide lines for constructing supergravity duals of $\mathcal{N} = 1$ gauge dynamics

So far we have investigated the realization of properties of $\mathcal{N} = 1$ gauge theories mainly in the backgrounds of KS and MN. We now would like to examine how such properties constrain the construction of any supergravity background dual to a confining $\mathcal{N} = 1$ gauge theory, such as the general class proposed in [53]. In particular we analyze the following properties: (i) Gauge group, (ii) Supersymmetries, (iii) Wilson loops ('t Hooft loops) and the corresponding quark anti-quark (monopole anti-monopole) potential, (iv) Instantons and the $\mathrm{U}(1)_R \to Z_{2N}$ symmetry breaking, (v) gluino condensation and the spontaneous $Z_{2N} \to Z_2$ breaking, (vi) monopoles, (vii) domain walls, (viii) baryons and (ix) KK states. This discussion does not constitute a set of restrictive rules but rather certain insights based on the experience gained from the analysis of existing models.

- In the original AdS/CFT duality the number of colors N in the boundary $\mathrm{SU}(N)$ gauge theory is associated with the number of the D3-branes or the flux of the 5-form. To have $\mathrm{SO}(N)$ or $\mathrm{SP}(N/2)$ gauge groups required an orientifold operation that replaces the S^5 with an $\mathbb{RP}^5 = S^5/\mathbb{Z}_2$ [52]. In orbifold models one mods with a discrete group, $\Gamma \in \mathrm{SU}(4)$. If the set of irreducible representation of Γ is $\{r_i\}$ with corresponding dimension d_i, then starting with a gauge group $\mathrm{U}(\sum_i d_i N)$ one finds after the orbifolding $\prod_i \mathrm{U}(d_i N)$. For an abelian discrete group the gauge group is $\mathrm{U}(N)^{|\Gamma|}$ where $|\Gamma|$ is the order of the discrete group Γ and the gauge group before orbifolding is $\mathrm{U}(\Gamma N)$. In particular for $\Gamma = Z_k$ one gets $\mathrm{U}(N)^k$. Another mechanism to generate holographic gauge groups is to consider supergravity backgrounds based placing D3-branes on conifold singularities. The original model based on $T^{1,1}$ [5] was generalized to ADE conifolds [54]. The gauge theories dual to both the orbifolds and the conifolds include matter superfields in bi-fundamental representations. To build a supergravity dual to a theory with a single SU(N) group factor (in $\mathcal{N} = 1$), wrapped 5-branes seem to be useful either following the lines of the MN model, or as a result of a Seiberg duality cascade as in the KS model. To get a single SO(N) or SP($N/2$) groups an orientifold will be needed.

- At least four mechanisms for breaking the maximal 16 supersymmetries down to the 4 supersymmetries of $\mathcal{N} = 1$ were suggested: (i) Orbifold models constructed from D3-branes at R^6/Γ orbifold singularities where $\Gamma \in$ SU(3) [55] (ii) Conifold models based on D3-branes at the singular point of a conifold with a base like the $T^{1,1}$ [5, 8] (iii) Wrapping the little string theory on S^2 and embedding the spin connection of the S^2 in a particular way in the SO(4) isometry group of the NS5-brane solution [10]. (iv) A soft breaking via a Myers mechanism [56] due to incorporating a RR magnetic 3-form in the AdS background [4, 57, 58]. For duals of superconformal gauge theories (i) and (ii) may serve as a starting point. To construct supergravity duals of field theories with soft breaking down to $\mathcal{N} = 1$ one can use (iv). For theories that in the IR resemble the $\mathcal{N} = 1$ SYM fractional branes should be added to the orbifold and conifold models or one can use the twisting approach of (iii). In fact it turns out that the two approaches (ii) and (iii) are closely related. It is easily shown that like in the MN model, in the conifold model of [5], the spin connection along the S^2 is identified with the $U(1)_R \in SU(2)_R$. Further more, an interpolation between the KS model and the wrapped D5-brane solution of MN was written down in [17]. These cases were shown to be special cases of an ansatz that describes spaces with topology $\mathbb{R} \times S^2 \times S^3$. The general conditions that a background of the form $R^{1,3} \times M^6$ has to be obey in order to preserve four supersymmetries are [59, 60, 17] (i) M^6 should be a hermitean manifold, (ii) the connection with torsion should have its holonomy contained in SU(3), (iii) The Kähler form and the complex structure should obey the dilatino Killing spinor equation. To construct $\mathcal{N} = 1$ models one may use one of the conifolds of Calabi-Yau compactification, or the method of fractional branes for instance with 5-branes wrapping two cycles other than the S^2.

- Models dual to a gauge theory in the confining phase have to admit Wilson loops with area law behavior [61]. As discussed in section 2, for cases with no additional 5-branes, this implies that either (i) $f(\tau_{\min}) > 0$ or that (ii) $f(\tau_{\text{div}}) > 0$ [23]. In models with additional 5-branes as part of the background, the world-volume physics on these branes should also be invoked [51]. Since F1-strings cannot end on NS5 branes, but D1-branes can, localizing such branes as part of the background may induce confinement. For models without such branes the question is what option out of the two (with $\tau_{\rm min}$ or with $\tau_{\rm div}$) is preferable. If in option (i) $\tau_{\rm min}$ is not the minimal value of τ then there might be an additional 4d boundary at a lower value of τ. In that case one has to check that signals cannot propagate between the two boundaries [62]. On the other hand option (ii), that is associated with a horizon, may result in a divergent string tension [31]. As was explained in section 2, any generic solution for which the string that corresponds to the Wilson loop is mostly a flat string along $\tau_{\rm min}$ (or $\tau_{\rm div}$), admits the desired features of the flux tube similar to those derived in flat space time. The counterpart of a confining Wilson loop should be a screening 't Hooft loop. Replacing the F1 connecting the external quark anti-quark pair with a D1 attached to a monopole anti-monopole system (or wrapped D2 in IIA) properly describes a 't Hooft loop in certain models [31, 33, 32]. Localized NS5-branes which are part of the background will induce screening behavior since the D1-branes can end on them. For backgrounds without such branes, since the action of the D1 strings is related to that of the ordinary strings by $e^{-\phi}$, such a construction can produce screening only provided that $e^{-\phi(\tau_{\rm min})}$ vanishes (or similarly with $\tau_{\rm div}$). This requires that the string coupling diverge at some point along the radial coordinate and hence it cannot serve as the supergravity dual mechanism of screening of monopoles. In type IIB performing S duality is not helpful here since it will transform the system to that of an F1. Instead one can attribute the 't Hooft loop to a fractional D1 that corresponds to a wrapped Dp-brane over a compact $p-1$ cycle. This mechanism is viable provided that the volume of the cycle vanishes at some value of the radial coordinate.

- To account for the gauge theory instantons, one may use D(-1) brane probes or wrapped euclidean Dp-branes on $p+1$ cycles, or some combination of the two. From the relation to the gauge coupling, the breaking of the $\mathrm{U}(1)_R$ symmetry and the existence of N degenerate vacua, in the MN and KS models, one can deduce a supergravity scenario of $\mathcal{N}=1$ instantons that includes the following ingredients: (i) The imaginary part of the action of the wrapped branes is proportional to N times the angle whose shift transformation corresponds to the $\mathrm{U}(1)_R$. In this way, the requirement that the change of the action under the symmetry transformation is a multiple of 2π, restricts the shifts only to elements of Z_{2N}. This means that the angle discussed has to be trivially fibered over the cycle the brane is wrapping. (ii) The real part of the action should be proportional to g_{YM}^{-2}. (iii) The cycle which is wrapped collapses to zero in the region that corresponds to the IR. Since the flux on a collapsing cycle has to be multiple of 2π, non-singular solutions can be achieved when the flux in the UV is a multiple of 2π and there is no change in the flux as a function of the radial direction. Combined with the previous ingredient that the flux is linear in the angle, and N, this requirement selects a set of only N values of the angle and hence N degenerate vacua. For instance one may imagine D0-branes of type IIA with euclidean time direction that wrap an S^1. The real part of the action is proportional to the circumference of the circle. The imaginary part to N times an angle that is perpendicular to the S^1. The radius of the S^1 should vanish in the IR.

- In $\mathcal{N} = 1$ models, in which gluino condensation is expected, there should be some complex supergravity mode (in general a mix of modes) that approaches the boundary in such a way that implies that the dual operator has a non-zero VEV. In the KS model this combination of modes reduces in the UV to one of the polarizations of C_2. In a general background it is impossible to guess what the dual supergravity mode will be, and one must calculate on a case by case basis. In [64] finite temperature generalizations of the KS and MN backgrounds were found. In these solutions there is no chiral symmetry breaking and therefore no gluino condensate.

- A probe configuration that can be associated with the $\mathcal{N} = 1$ domain wall is a wrapped Dp-brane on a $p-2$ cycle ($p > 3$) that admits BPS solutions. If the background is a function of $g_s N$ (and not of N separately, then in the large N limit the domain wall tension is necessarily linear in N, since the action of a Dp-brane is proportional to ${g_s}^{-1} = N\lambda^{-1}$. Similar to the construction in MN and KS the interpolation between two vacua that differ by a phase of $\frac{2\pi i k}{N}$ is associated with a group of k wrapped branes. The dual of the flux tube, an F1 string connecting two points on the boundary, can end on a single wrapped Dp-brane; whereas on a stack of k such branes only k coincident strings can end. This is due to the fact there is a 3d U(k) gauge theory on the k domain walls and the end of a string on this stack of branes is a quark in the fundamental representation. In fact, if the 3d $\mathcal{N} = 1$ U(k) gauge theory has a Chern-Simons term [49], the above statement has to be reexamined.

- The role of baryons can be played by wrapping of Dp-branes over p-cycles. Baryon vertex connected to N external fundamental quarks is dual to such a wrapped brane which is necessarily connected to N strings to conserve the charge associated with the N units of flux. As was shown in section 4 such baryons have mass which is N times that of the corresponding "mesons". Baryons composites of elementary particles that are incorporated in the background (as oppose to external ones) can be constructed from wrapped branes connected with strings between themselves and not with the boundary. In particular if on top of the wrapped Dp-brane there are also Dp'-branes on p' cycles then one can form objects made out of k wrapped Dp-branes connected with

$kN_p = k'N_{p'}$ strings to k' wrapped Dp'-branes. Since in $\mathcal{N} = 1$ gauge theory the baryons cannot be BPS states, the wrapped branes should also have no left over supersymmetries.

- A common feature to all the supergravity backgrounds dual to $\mathcal{N} = 1$ gauge theories is the KK modes that have masses of the same order of magnitude as the glueballs. To decouple these states one needs to be in the region where $\alpha'\mathcal{R} \sim (g_s N)^{-1/2} \gg 1$, namely, in a region where the curvature is large and the supergravity approximation is not valid. The question is whether one can get rid of these modes without destroying the desirable features of the models. One alternative to avoid the excitations associated with the S^2 and S^3 cycles is to have a non-critical model without the extra compactified dimension. A possible mechanism for that may be the replacement of the conifold by some CFT model like the $c = 1$ super Liouville model [63] that should obviously also incorporate RR fields. The problem with this approach is two folded: (i) to write down the appropriate CFT model (ii) to find a way to mimic all the non-perturbative properties associated with the wrapping of branes over these cycles. Another alternative is to have two scales in the problem. One big mass scale associated with the compact cycle which is the host of the wrapped branes, and another scale corresponding to the QCD scale which is not characterized by a compact cycle but rather by a distance between non compact branes like in MQCD.

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